Understanding Power, Torque and Gearing

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Slide 1: To understand Power and Torque, you must first understand force and its relationship to mass, velocity, acceleration, and the “work” done by the force over time.

Acceleration: The rate of change of velocity of something. If the velocity of a body increases by 2 meter per second, every second,, its acceleration is 2m/s/s or 2m/s^2.

Force: Is the push or pull that will cause an unconstrained body to accelerate and is measured in Newtons, or N. Force = mass × acceleration. A force of 10 newtons will cause a 5kg body to accelerate at 2m/s^2: 5kg × 2m/s^2 = 10N. Therefore Newton = kg m/s$^{2.}$

Forces are cumulative. If a body has a 15N force acting on it to the left (say a man pushing it) and a 7N force acting to the right (say the force of friction), the net force is $15N_{(left)} - 7N_{(right)} = 8N_{(left)}$, or 8N to the left. If it has a mass of 2kg, it will accelerate to the left according to a=F/m = 8N/2kg = 4m/s^2.

If the net forces on a body is zero, it will cease to accelerate but if in motion, will stay in motion. If you are applying 15N to a body, and its velocity of 2m/s causes friction of 15N to be generated, the body will continue to move at 2 m/s, with acceleration = 0 since the net force is zero. If you stop applying the 15N force, the 15N of friction force will cause negative acceleration, or deceleration, and the body will come to a stop. So net positive force force causes acceleration, net negative force causes deceleration.

Work: Is the effective application of force. If you apply a force to an object that does not move, no work is done, no energy is transferred, nothing happens. So if you push a wall that does not move, you have done no work. Work = Force applied × the distance over which the force is applied. The unit of work is Joule. So, if you apply a 5N force to an object that moves 3meters as a result while that force is applied, the work done = Force × distance = 5N × 3m = 15Nm = 15 Joules. So 1 Joule = 1Nm.

Very importantly, the unit of energy is also Joule. If you have done 1 Joule of work, you have involved the transfer of 1 Joule of energy.

Power: Is the *rate* of doing work, thus the rate of the transfer of energy. Power = Work / Time. The unit is watt (in the SI system, horsepower in the imperial system). So from the above example, if the force of 5 newtons is applied over a distance of 3 meters, for a time of 2 seconds, power = work/time = force × distance / time = 5N × 3m/ 2 seconds = 7.5Nm/s = 7.5 watts.

Now this is very important: In our example, moving the object that distance in that time took 5Newtons and consumed 7.5joules per second of energy or 7.5 watts. To have 7.5watts means you have the CAPACITY to expend 7.5Joules of energy per second, or do 7.5 Newton meters of work per second

So we now know that power = F × d/t. Since d/t = velocity, we know that power = Force × velocity and also, Power = energy consumed/ second.

Slide 2: What is torque and how does it relate to power?

Torque: Is *rotational force*. Torque = Force × Distance from axis of rotation. When you tighten a nut on your wheel, you are applying torque. In a car engine, torque is the actual physical rotational force that the engine generates If your wheel spanner is 0.5m long and you apply 7N to the end, you are applying a torque of 7N × 0.5m = 3.5Newton meters, or 3.5 Nm (note the units of torque and work are the same, even though they are not the same thing).

Torque multiplies and and rotational speed divides with gearing, and vice versa. So if your gear ratio is 2:1, and you apply 7Nm of torque at a rate of 1 revolution per second, you will get an output of 14Nm at a rate of 0.5 revolutions per second. If your gearing is 1:2, you will get 3.5Nm of torque output at 2 revolutions per second

Relating Power and Torque: We saw that power is the application of force over a distance in a certain time period, or the *rate* of application of force over a distance. If you convert your frame of reference from linear (straight line) to rotational, the same concept applies. You tighten your lug nut with 7N of force with a 0.5meter spanner which turns at the rate of, say 0.5 revolution per second. What's the power you are expending? Remember that Power = Force × distance/time. The force is applied around the circumference of the circle that the spanner rotates, $2\pi r$. the time it takes to cover that distance at 0.5 revolutions per second = 2 seconds. So, Power = $F \times 2\pi r/t$ = 5N × 2π × 0.5m/2 seconds = 11Nm/s = 11 watts.

We can express power as a function of torque and rotational speed instead of force and radius. $P = F \times 2\pi \times r/t$. We can rearrange this so $P = F \times r \times 2\pi/t$. $F \times r$ = Torque and $1/t$ = Revolutions per second (RPS), we can rewrite this equation as Power = Torque × 2 π × RPS. We know the torque is 3.5Nm. We know rotational speed is 0.5RPS. So Power = 3.5Nm × 2 π × 0.5/second = 11Nm/s = 11 Watts.

Also RPS = RPM/60.

Therefore Power (watts) = Torque × 2 π × RPM/60. Since 1hp = 746watts, Power (hp) = Torque (Nm) × 2 π × RPM/(60 × 746) or,

Power (hp) = Torque (Nm) × π × RPM / 22,380

Slide 3: What it all means.

So what does it all mean?

-Torque is rotational force the engine generates. After the gearing ratio effects of the transmission and differential, you get the final torque at the wheels. That torque, divided by the diameter of the tyres, gives the FORCE at the contact patches that propels the car.
-There are forces that oppose the motion of the car. Friction at the road surface, the internal distortion of the tyres, air resistance, friction in wheel bearings, transmission and differential gears, etc. The engine generated force minus the resistance forces leaves the net force. The car's acceleration is directly proportional to this force.
-As the car moves, energy extracted by the engine from the fuel is consumed. It is dissipated as heat in all the places where friction occurs, it is transferred to the air when the air is pushed out of the way, and it is absorbed by the car itself as Kinetic energy.
-The rate of energy produced by the engine per second is the power produced. The rate consumed is the power consumed. Whatever is left over is lost as waste heat from the engine cooling system, or can be used to accelerate the car to a higher speed.

-Torque then, is the force of the engine that causes the car to accelerate. Some of the force is used up to counter the resistant forces (such as friction and air resistance), the rest produces acceleration

-Power is the rate at which energy is transferred to the air as heat or motion when the forces of the engine are applied against friction and air resistance.

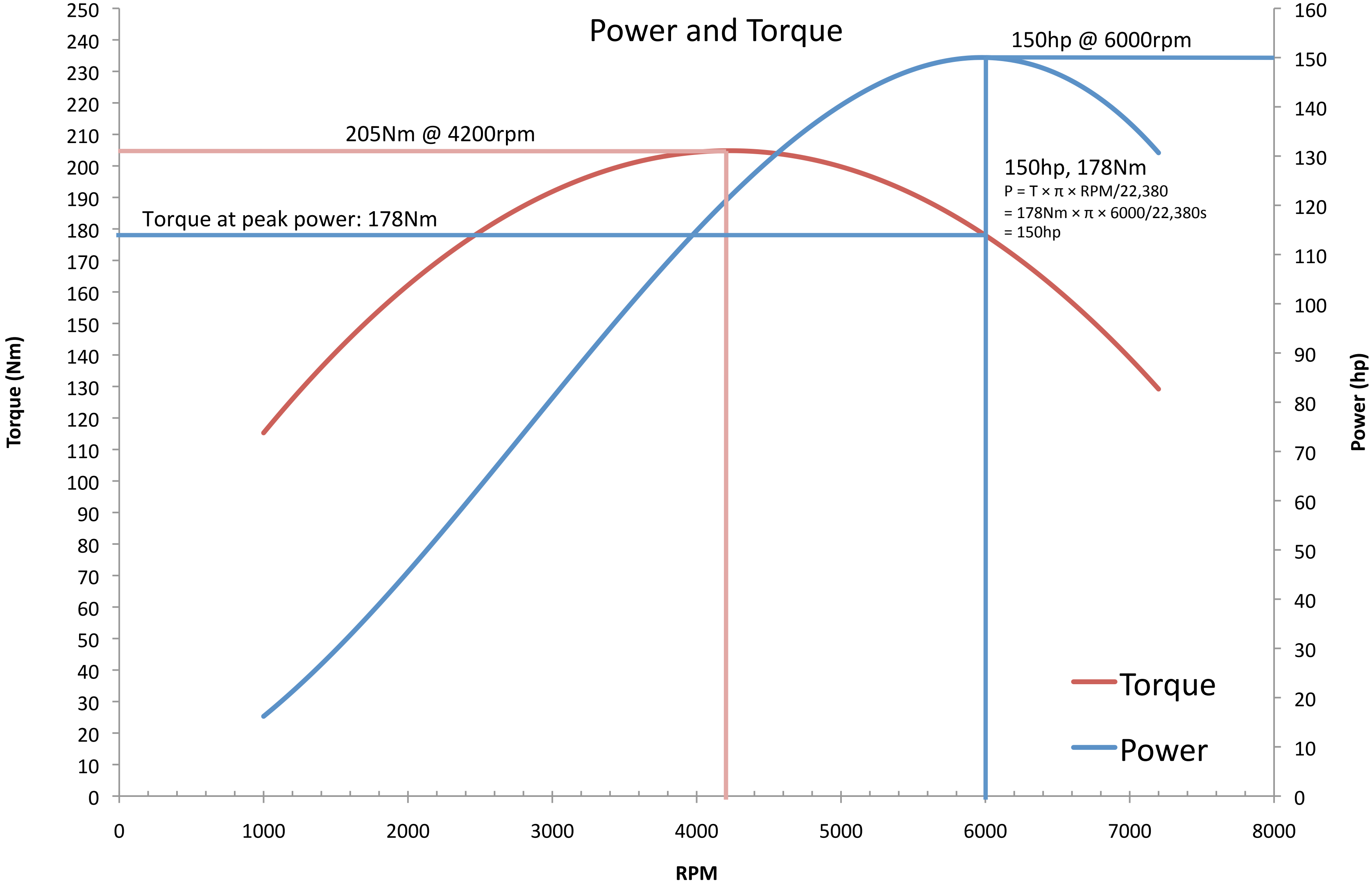
-Power determines how fast a car can go because how fast a car can go is limited by how much energy it can produce per second to overcome the energy its friction and drag dissipates per second.

-At any speed, the max acceleration you can get is determined by how much force (or torque) you have available after that which is used up countering the resisting forces (friction and air drag). If a 1500kg car produces 3,000 Newtons of force at the contact patches at 120km/h, and faces 2000Newtons of resistance, you have 1,000Newtons available to accelerate the car. $F = m \times a$, or $a = F/m = 1000N/1500kg = 0.67\ N/kg$. Since $1N = 1kg\ m/s^2$, $a = 0.67\ (kg\ m/s^2)/kg = 0.67 m/s^2$.

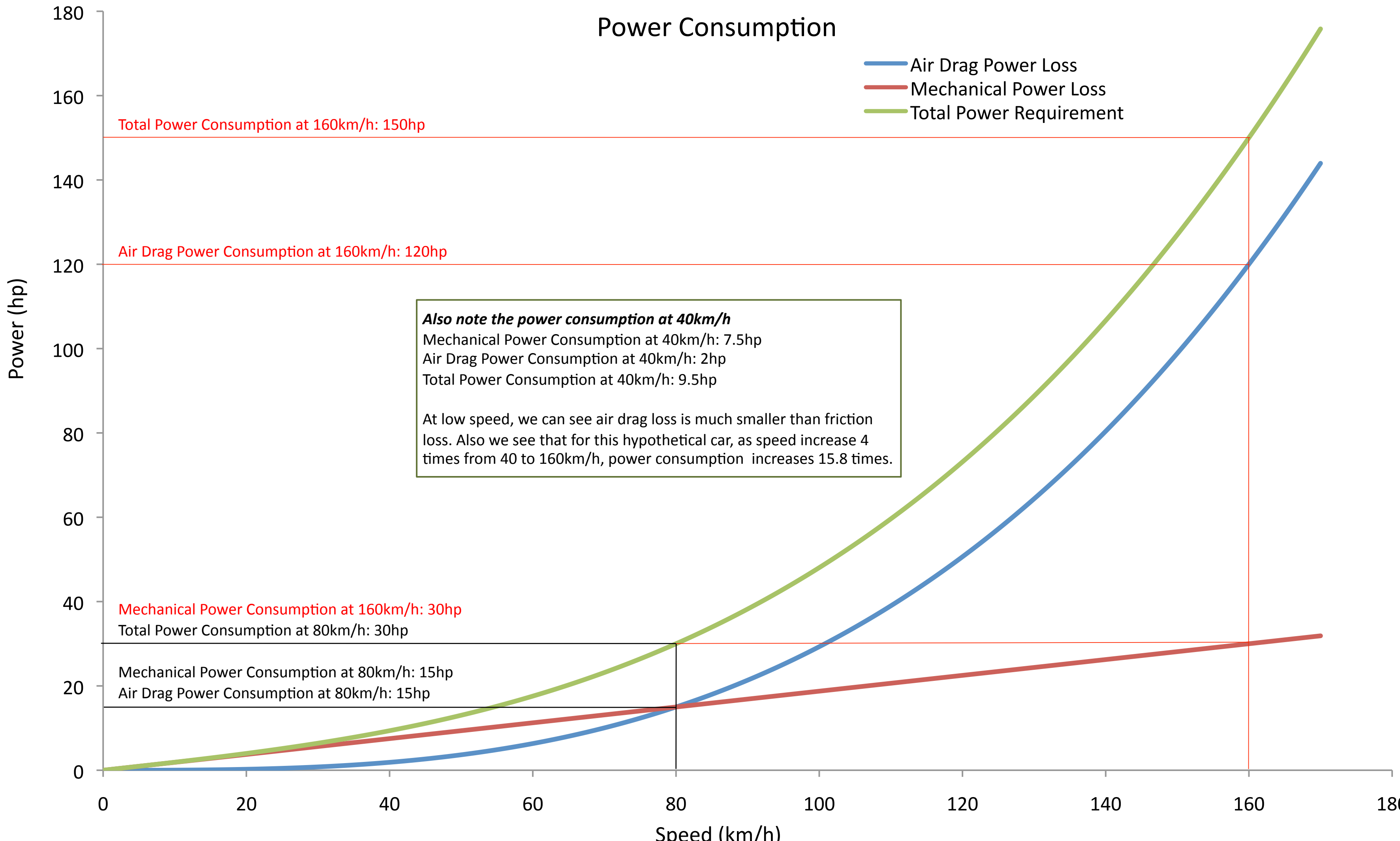
-Alternatively, the max acceleration you can get is determined by how much more energy per second you have available in excess of that being used up in overcoming friction and air drag. The car produces 100kW at 100km/h. It consumes 67.7kW at that speed, leaving 33.3kW available. We know that $P = F \times v$ and $F = m \times a$. Therefore, $P = m \times a \times v$. Rearranging that equation we get acceleration $a = P/(m \times v) = 33{,}333watts/(1500kg \times 33.33m/s)$. $= 0.67\ watts/(kg\ m/s) = 0.67\ (N\ m/s)/kg\ m/s) = 0.67N/kg$ $=0.67\ (kg\ m/s^2)/kg = 0.67\ m/s^2$.

Conclusion: A car produces a force at the contact patches due to the torque produced by the engine , amplified (or reduced) by the gearing, and converted into linear force the contact patches according to their circumference. A car's acceleration is determined by its mass and how much force it can create at the contact patches in excess of the force required to overcome the resisting forces. When resisting forces reach the force created by the engine, the car ceases to accelerate. Alternatively, a car's acceleration is determined by how much excess power it has available above and beyond the power being expended at the current speed to overcome resisting forces.

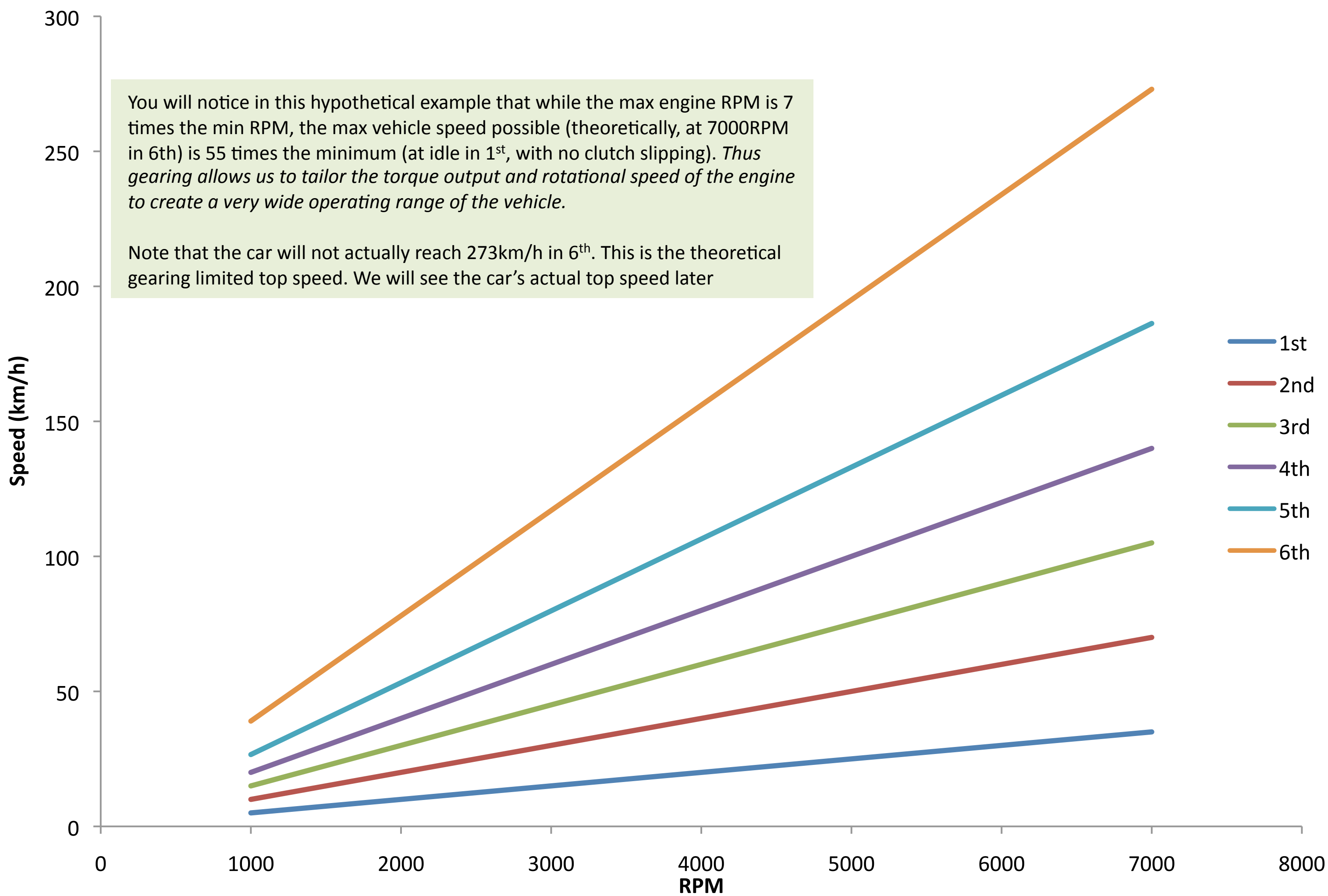
Slide 4: We will apply the theory to a hypothetical car and see what power, torque, gearing, air resistance, friction, all mean for acceleration and speed. These are the power and torque curves of our hypothetical car. The Power and Torque at any RPM are related according to Power (hp) = Torque (Nm) × π × RPM/22,380. This relationship means that because RPM keeps rising, even as the torque falls off, the power keeps rising up to a point.



Slide 5: Here we model the Power consumption by speed of our hypothetical car. The two main components are power consumed to overcome air resistance and to overcome mechanical friction (driveline and rolling resistance). Aerodynamic drag increases as the square of speed. This means that as speed doubles, the aerodynamic drag quadruples. Since Power = F × v, power consumption of aerodynamic drag increases as the cube of speed, so twice as fast means $2^3 = 8$ times higher power. Mechanical loss is dependent on transmission rotational speeds as well as vehicle speed, but for simplicity we model it as directly related to vehicle speed, so twice as fast means twice the mechanical power consumption. At low speeds, most power consumption is mechanical (red line in the graph). As speed increases, drag power consumption (blue curve), because it cubes with speed, increases drastically. The sum of the drag and friction power gives us the total power consumption (green curve) of our hypothetical car, which we will use to model the performance

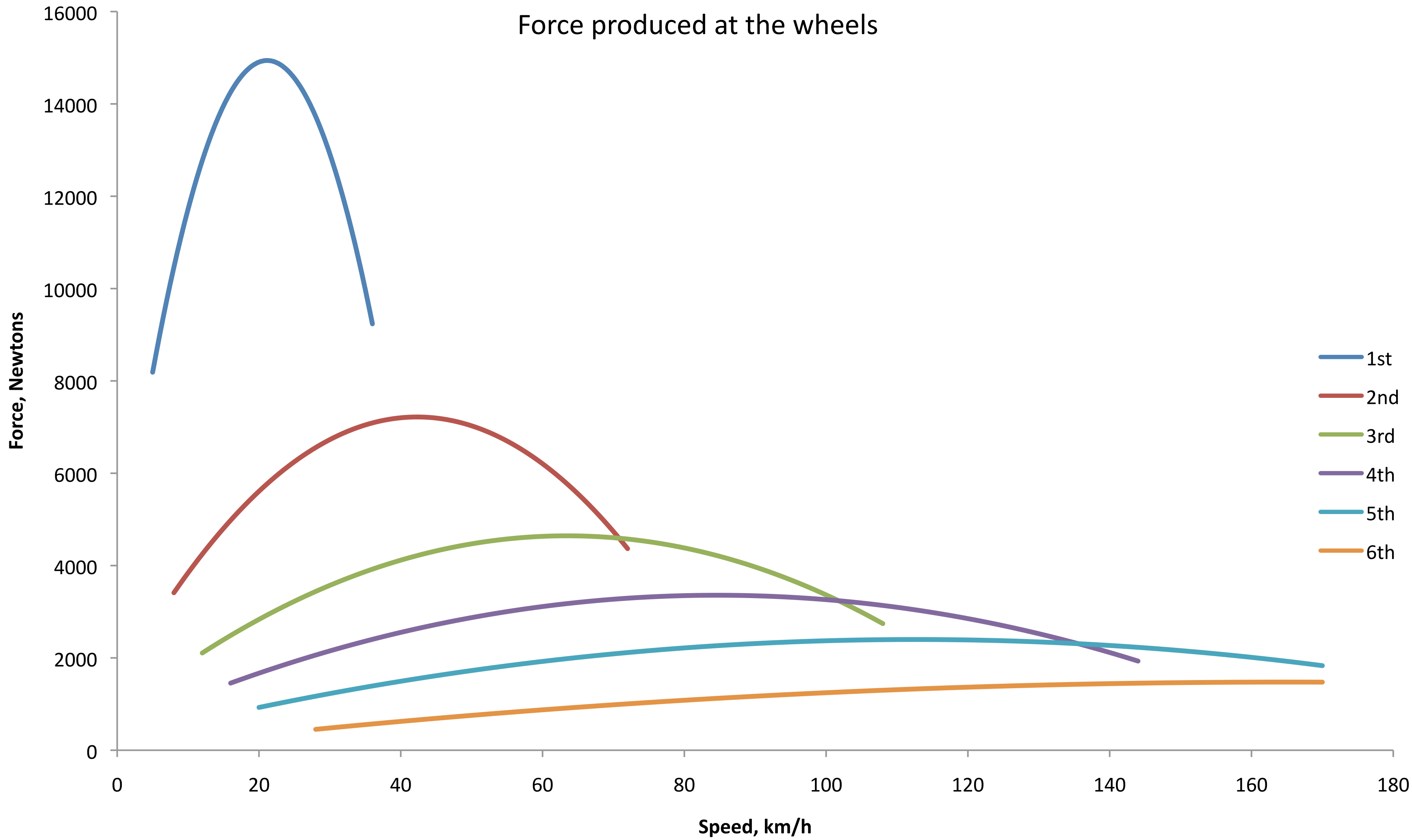


Slide 6: This graph is the hypothetical car's gearing. Gearing is used to multiply torque and RPM. A 2:1 gearing ratio will multiple torque by 2, while reducing RPM by 2. An overdrive gear, say 0.7:1 will reduce torque and increase rotational speed. Keep in mind that total gearing effect is the difference between engine RPM and wheel RPM, which the result of gearbox ratios and differential ratio. In our hypothetical car 4th is 1.33:1 and final drive is 4.59:1. So total gearing in 4th is 1.33 × 4.59 = 6.1:1. So at 1000rpm of the engine, the wheels will turn at 1000/6.1 = 163.9rpm. The hypothetical car has 205/65R15 tyres with a circumference of 2.0meters. So with every revolution of the tyre, the car will travel 2 meters. Therefore, in 4th, at 1000rpm, the car will travel 163.9rpm × 2m/rev × 60minute/hour × 1km/1000m = 20km/h. So 4th is geared at 20km/h per 1000rpm. The table gives the gearing for all the ratios. In reality, 2,3, and 4 would not so evenly spaced and 6th wouldn't be so tall)

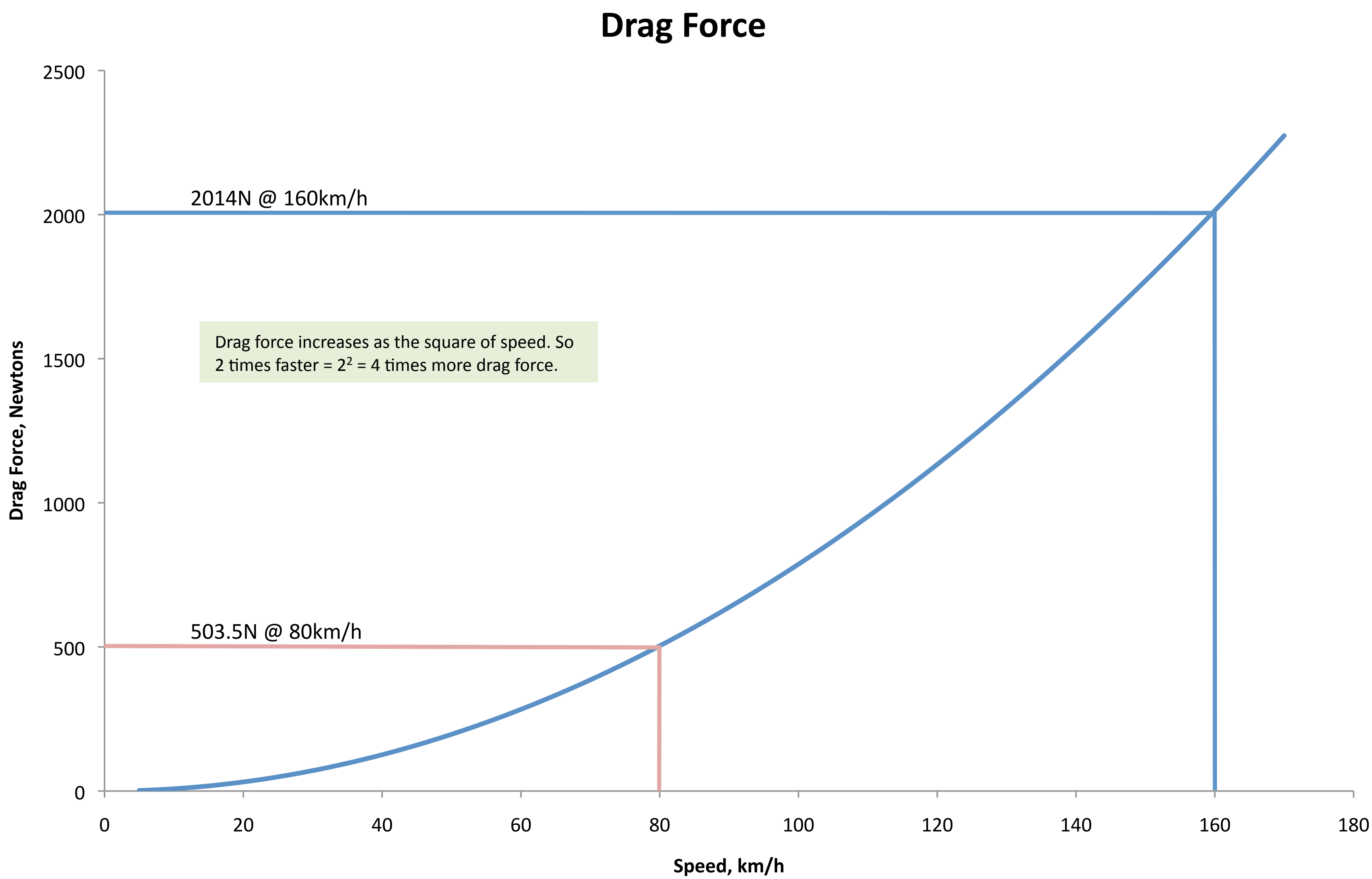


Gear	Vehicle Speed per 1000 engine RPM
1st	5 km/h
2nd	10 km/h
3rd	15 km/h
4th	20 km/h
5th	26.6 km/h
6th	39 km/h

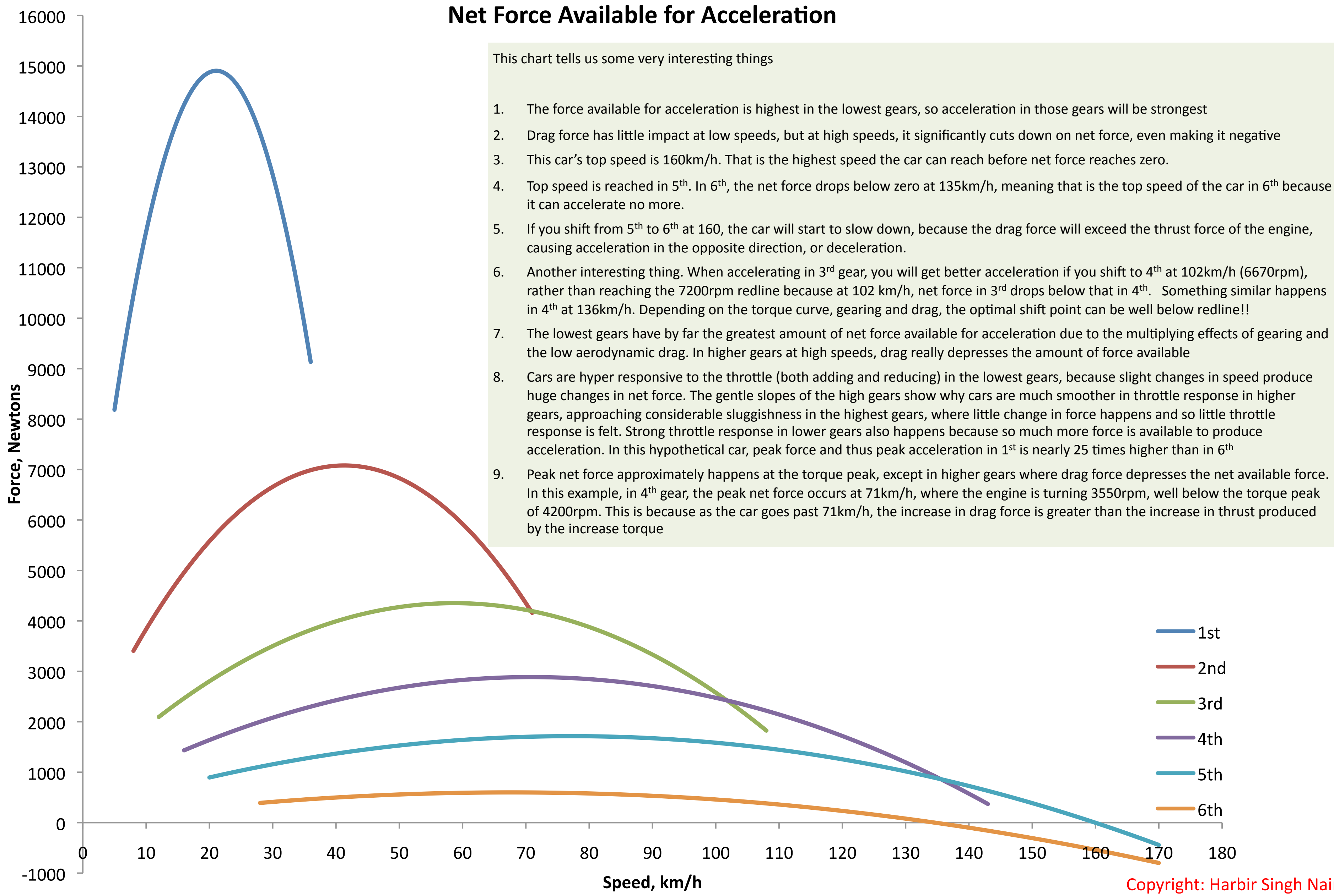
Slide 7: Gearing determines the torque at the drive wheels and thus the actual forward thrust generated. The Torque = force × distance from axis of rotation, the force the tyres exert on the road is torque at the wheels/tire radius. This force is the thrust to accelerate the car, as well as the force to overcome air drag. Contact patch force is net, meaning it's the force that remains after subtracting out the forces of friction in the transmission, differential, wheel bearings, and tyre sidewalls. Since Power = F × v, knowing the velocity and power loss, we can work out the the net force transmitted at the contact patches. $F_{contact\ patch}$ = [(Engine Torque × total gearing ratio) /tyre radius) – (mechanical power loss /velocity)]. Since we have assumed that mechanical power loss is related directly to vehicle speed, we end up with a constant friction force in the driveline (of about 500 N) regardless of speed. Its not perfectly accurate but driveline loss is too complex to model more accurately here.



Slide 8: Now we come to aerodynamic drag force. As we have said, a car travelling at speed encounters air resistance. At steady speed, the force at the contact patches pushing the car forward is exactly balanced by the drag force. To accelerate, a car needs to create a larger thrust than the drag force at that speed, and the excess is what is available to produce acceleration. Now, we modeled on slide 5 the power dissipated due to air resistance by speed. This lets us work out the drag force on the car at all speeds, according to F=P/v.



Slide 9: Now we're getting some place. The net force to produce acceleration is the difference between the thrust produced at the contact patches and the drag force on the vehicle. The greater this force, the greater the acceleration possible. When net force is zero, no acceleration is possible. When it is less than zero, drag force exceeds thrust and the vehicle will slow down! This slide shows for each gear, the net force (thrust – drag) by speed



Slide 10: This graph shows the power produced in 4th, 5th and 6th gears, against the car's total power requirement (drag + mechanical). We wee that 4th is well above the required power, and thus will produce the strongest acceleration (Net positive force = Excess power ÷ velocity), but the car runs into redline in 4th before reaching its potential top speed. What is potential top speed of the car? The speed at which the max power output equals the power demand. We have 150hp peak, and the car has a 150hp requirement to travel at 160km/h. That is its potential top speed. The gear ratio of 5th is selected so that the car can reach the speed where the power demand equals the power output, 160km/h @6000rpm. In 6th, power requirement exceeds power supply at 135km/h, and the car can accelerate no further because it has no more power available and can not increase the thrust at the contact patches to overcome higher drag forces.